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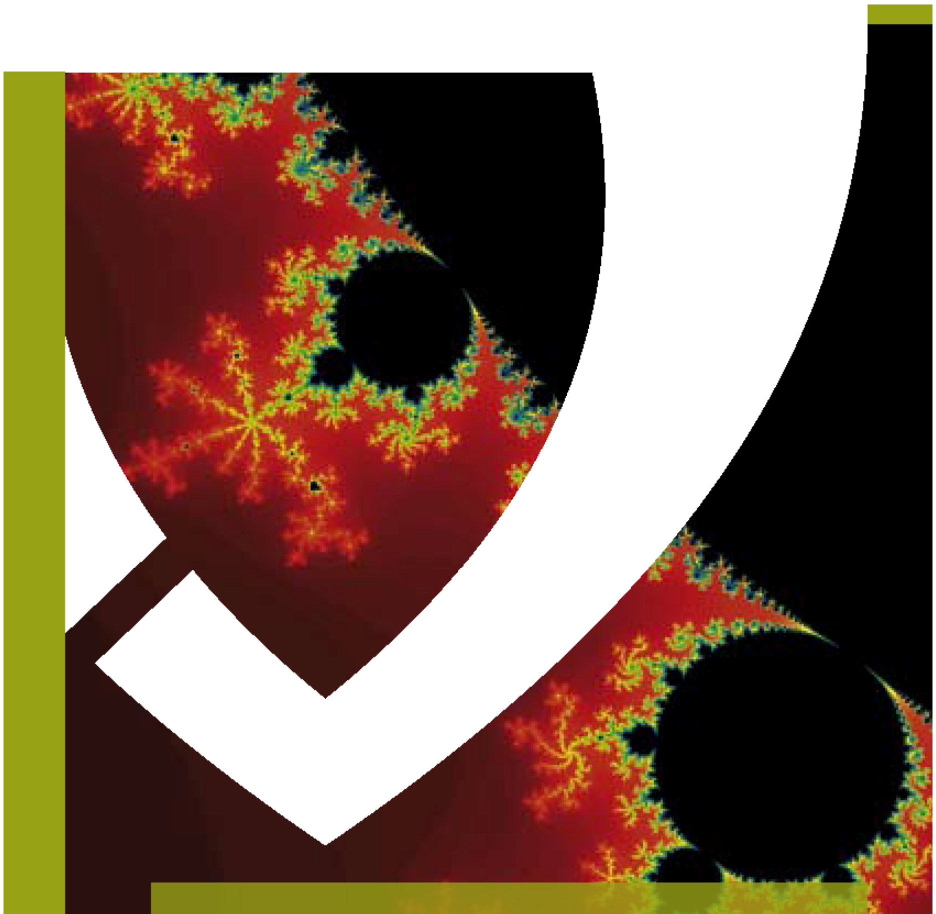

School of Engineering and Science

Mathematical Sciences

Graduate Program

Contents

1 Concept

1.1 General Description

The Mathematical Sciences Graduate Program at Jacobs University offers the opportunity for graduate study in pure, applied and computational mathematics, as well as in mathematical physics. The program leads to a *doctorate degree (PhD)*; a *Master's degree (MSc)* may be obtained as well.

This is an "integrated PhD program" which accepts students holding a Bachelor degree, as well as more advanced students. An early beginning has the advantage that students can spend their first semesters in the program exploring research areas and meeting possible advisors before having to finalize their choice, thus making better informed decisions. More advanced students are admitted at a level compatible with their previous education.

The initial part of the program involves a broad education in mathematical science, followed by a choice of advanced courses, seminars, and research activities leading to a dissertation.

Graduate students at Jacobs University are viewed as professionals. From early on, they are integrated into the faculty's international research collaborations, they routinely participate at international research conferences or in longer thematic research programs—a head start into a successful career in academia or industry.

1.2 Program Overview and Duration

Doctor of Philosophy (PhD) Students entering the graduate program with a Bachelor degree are required to complete successfully up to three semesters of coursework and a qualifying exam before progressing to the PhD dissertation. The program generally takes up to five years after the BSc degree. A separate MSc thesis is not required for students working towards a PhD degree, but students have the option to earn a separate Master's degree en route.

Students holding a Master's degree (or equivalent) typically need no more than three years until completion of their PhD degree.

Master of Science (MSc) The MSc degree requires up to three semesters of full-time coursework and one semester to produce a Master's thesis.

1.3 Interactions

Members of the Graduate Program in Mathematical Sciences interact with many faculty members and programs within the School of Engineering and Science, within Jacobs University at large, and with researchers worldwide. In particular, our weekly mathematics colloquium brings in leading mathematicians from Europe and overseas in all areas of mathematical sciences, in addition to the regular research contacts of our faculty members.

Moreover, graduate students with interests in applied, numerical, or computational mathematics are supported by Jacobs University's Computational Laboratory for Analysis, Modeling, and Visualization (CLAMV). CLAMV is equipped with advanced graphics workstations, a Linux cluster, a Sun Fire compute server, and has access to the Northern German supercomputing network. Jacobs University offers many opportunities for interaction with researchers in other fields—including geophysics, astrophysics, computer science, physics, psychology, neurosciences, and social sciences—whose work involves mathematical modeling and computation.

Graduate students with interests in Mathematical Physics can benefit from the course offerings, seminars and research activities of the Astroparticle Physics Graduate Program at Jacobs University. Traditionally there has been a strong cross-fertilization between mathematics and physics. Mathematics provides the language and forms the foundation of modern physics. Physics has inspired many important developments in mathematics. More than ever this is true today. Graduate students who want to do research in modern mathematical or theoretical physics need a strong mathematical background as it is provided in our graduate program.

1.4 Career Options

The graduate program in mathematical sciences at Jacobs University is designed to equip students with the necessary tools and scientific maturity to embark on a research career in academia or industry. Due to the central role of mathematics in science, there is a never ceasing demand for mathematicians in academia worldwide. Universities and colleges offer tenure-track and tenured positions to PhDs; certain positions are more focused on research and others more on teaching. Graduates in mathematical sciences are well sought after by non-academic employers. Consequently, mathematicians enjoy a large choice of well-regarded jobs outside of the university world, for example in research and development, finance, banking, and management.

2 Structure of the Program

Graduate education at Jacobs University is governed by the appropriate policies. Additional program specific rules are described below.

2.1 Initial Academic Advisor

Every incoming graduate student is assigned an initial academic advisor prior to coming to Jacobs University. The initial advisor guides the graduate students through the program, monitors his or her progress, and helps him or her select a PhD advisor.

2.2 Study Plan: Integrated PhD Program

The following study plan is the default variant for students entering with a BSc degree. Faster progress is always possible.

Semester	Coursework	Research	Additional Examinations
1–2	3 courses, 1 seminar		
3	3 courses, 1 seminar	Preliminary work	Qualifying exam must be completed by beginning of 4th semester
4	1 course, 1 seminar	PhD research proposal	PhD proposal must be presented by beginning of the 5th semester
5–9	1 seminar	PhD research	
10	1 seminar	PhD dissertation	PhD thesis must be defended by end of the 10th semester

An individual course plan is prepared by every graduate student in cooperation with his or her academic advisor and further faculty members as appropriate. Qualified students can enter the program at various advanced stages, depending on their qualifications. For instance, the graduate committee may waive the qualifying examination for students holding an MSc degree.

2.3 Course Requirements

In order to obtain a PhD or MSc degree, a student has to satisfy the following coursework requirements (in addition to the general Jacobs University requirements):

1. For a PhD degree: graduate courses and seminars worth at least 95 ECTS credits
2. For a Master's degree: graduate courses and seminars worth at least 95 ECTS credits
3. For both degrees: the courses Algebra (100 421), Real Analysis (100 411) and Complex Analysis (100 312 or 100 412).

Throughout their studies, graduate students are required to take one graduate level seminar each semester. In addition, all graduate students are expected to regularly attend the mathematics colloquium.

Graduate classes are 400 level courses and above; up to five 300 level undergraduate courses may be counted towards the course credit for PhD or Master's degrees.

Courses at 300 and 400 level carry 7.5 ECTS credits; graduate seminars generally carry 5 ETCS credits. A research proposal, including presentation, carries 25 ECTS credits. The Master's thesis carries 25 ECTS credits as well.

2.4 Qualifying Examination

Every graduate student working towards a PhD in mathematics must pass a comprehensive examination in mathematics before the beginning of the fourth semester. The purpose of this examination is to manifest solid knowledge of advanced but core material in mathematics, to show the ability to make connections between areas of mathematics usually taught in different courses, and to demonstrate the potential for research in the mathematical sciences.
The examination is oral, it takes at least 90 minutes and covers material described as follows:

1. Algebra
2. Real Analysis
3. Complex Analysis

plus a choice of two among the following topics

4. Topology
5. Numerical Analysis
6. Partial Differential Equations
7. Mathematical Physics
8. Functional Analysis
9. Probability Theory

The syllabi for these topics are presented in Section 4. The examination is given by three professors appointed by the graduate committee.

2.5 Master's Option

Any graduate student may at any time request to work for a Master's degree, independently of whether or not he or she continues to work for a PhD degree.
The graduation requirements are specified by the Jacobs University's policies; the coursework requirements are described in Section 2.3. There is no separate Master's examination or qualifying examination.
The following table describes a study plan for a graduate student who has entered the program with a Bachelor's degree and wishes to conclude his or her graduate education with a Master's degree after four semesters.

Semester	Coursework	Research
1–2	3 courses and 1 seminar	
3	2 courses and 1 seminar	Preliminary work
4	1 course and 1 seminar	Master's thesis

3 Courses

The graduate courses offered in the first three semesters of the graduate program in Mathematical Sciences have two purposes. The first is to provide a solid and broad foundation of mathematical knowledge that is needed for doing research in mathematical sciences. The second purpose (at least of some of the courses) is to also provide an introduction to a specific area of current research, so as to give the students some basis for their decision on their area of specialization.

3.1 400 Level Courses

100411 – Real Analysis

Short Name: RealAnalysis
Type: Lecture
Semester: 1
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents Real Analysis is one of the central courses in the advanced education of mathematics students. The course is centred on abstract integration theory and measure spaces. The discussion of spaces of integrable functions will lead to a discussion of Hilbert and Banach spaces. Some of the central results of Functional Analysis, e.g., the Hahn-Banach theorem and the open mapping theorem will be proven.
Due to the central role of integration in applied sciences, this course should also attract ambitious physics and engineering undergraduate and graduate students.

100412 – Complex Analysis

Short Name: CompAnalysis
Type: Lecture
Semester: 2
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents Complex Analysis builds on the material taught in the undergraduate Complex Variables course. After a quick review of the most important results and concepts, some more advanced topics are covered. Possible subjects are Riemann Surfaces, Elliptic Functions and Modular Forms, Complex Dynamics, Geometric Complex Analysis, or Several Complex Variables. Which subjects are chosen will depend on the instructor and on the students' interests. This course may also provide an introduction to a specific area of research, leading to possible PhD thesis projects.

100471 – Functional Analysis

Short Name: FunctAnalysis
Type: Lecture
Semester: 2
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents This course assumes basic knowledge of measure and integration theory, and of classical Banach and Hilbert spaces of measurable functions. Functional Analysis focuses on the description, analysis, and representation of linear functionals and operators defined on general topological vector spaces, most prominently on abstract Banach and Hilbert spaces. Even though abstract in nature, the tools of Functional Analysis play a central role in applied mathematics, e.g., in partial differential equations. To illustrate this strength of Functional Analysis is one of the goals of this course.

100421 – Algebra

Short Name: Algebra
Type: Lecture
Semester: 1
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents Advanced topics from algebra, including groups, rings, ideals, fields, and modules, continuing the course Introductory Algebra (100 321).

100422 – Advanced Algebra

Short Name: AdvAlg
Type: Lecture
Semester: 2
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents This course develops more advanced topics in algebra beyond those from the Algebra course (100 421), including commutative and non-commutative algebra (and their relations to algebraic geometry), categories and homological algebra, and representation theory.

100431 – Number Theory

Short Name: NumberTheory
Type: Lecture
Semester: 3
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents This course is mainly an introduction to algebraic number theory, but it also covers some analytic number theory, most notably the Dedekind zeta function and the analytic class number formula. Topics include algebraic number fields and their rings of integers, ideal theory in Dedekind rings, localization, p-adic numbers and fields, ideal class group and unit group, finiteness of the class number, Dirichlet unit theorem, Dedekind zeta function, analytic class number formula, perhaps Dirichlet L-series and a proof of Dirichlet's theorem on primes in arithmetic progressions, Artin reciprocity with the main results (no proofs) of class field theory.

100442 – Algebraic Topology

Short Name: AlgebrTopology
Type: Lecture
Semester: 2
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents This course is mostly concerned with the comprehensive treatment of the fundamental ideas of singular homology/cohomology theory and duality. The knowledge of fundamental concepts of algebra as well as of general topology is assumed (at a level of Introductory Topology and Introductory Algebra).
The first part studies the definition of homology and the properties that lead to the axiomatic characterization of homology theory. Then further algebraic concepts such as cohomology and the multiplicative structure in cohomology are introduced. In the last section the duality between homology and cohomology of manifolds is studied and few basic elements of obstruction theory are discussed.
The graduate algebraic topology course gives a solid introduction to fundamental ideas and results that are used nowadays in most areas of pure mathematics and theoretical physics.

100451 – Differential Geometry

Short Name: DiffGeom
Type: Lecture
Semester: 3
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents Differential geometry is the study of differentiable manifolds. Assuming basic concepts from 100 311 (Integration and Manifolds) and 100 351 (Introductory Geometry), such as manifolds, differential forms, and Stokes' theorem, the focus in this course is on Riemannian geometry: the study of curved spaces which is at the heart of much current mathematics as well as mathematical physics (for example, General Relativity).

100452 – Lie Groups and Lie Algebras

Short Name: LieGroups
Type: Lecture
Semester: 2
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents A Lie group is a group with a differentiable structure, the tangent space at the identity element of a Lie group is its Lie algebra. Lie groups and Lie algebras are indispensable in many areas of mathematics and physics. As a mathematical subject on its own, Lie theory has led to many beautiful results, such as the famous classification of semisimple Lie algebras. In physics, Lie groups and their representations are essential to the theory of elementary particles and its current developments. Due to the close correspondence of physical phenomena and abstract mathematical structures, the theory of Lie groups has become a showcase of mathematical physics.
The course presents fundamental concepts, methods and results of Lie theory and representation theory. It covers the relation between Lie groups and Lie algebras, structure theory of Lie algebras, classification of semisimple Lie algebras, finite-dimensional representations of Lie algebras, and tensor representations and their irreducible decompositions.
A solid background in multivariable real analysis and linear algebra is presumed. Familiarity with some basic algebra and group theory will also be helpful. No prior knowledge of differential geometry is necessary.

100432 – Algebraic Geometry

Short Name:	AlgGeometry
Type:	Lecture
Semester:	2
Credit Points:	7.5
Prerequisites:	None
Corequisites:	None
Tutorial:	No

Course contents Algebraic geometry is the study of geometry using algebraic tools: the geometric objects are the common roots of a set of polynomials in several variables. Many geometric properties can be studied in terms of algebraic properties of these polynomials, using the powerful machinery of algebra to study geometry.
Basic concepts from 100 421 (Algebra) and 100 321 (Introductory Algebra) are used in this course. Among the studied subjects are affine and projetive varieties, schemes, curves, and cohomology.

100461 – Dynamical Systems

Short Name:	DynSystems
Type:	Lecture
Semester:	3
Credit Points:	7.5
Prerequisites:	None
Corequisites:	None
Tutorial:	No

Course contents Based on the undergraduate ODE/Dynamical Systems course, this course goes more deeply into the theory of discrete and continuous dynamical systems. Possible topics include bifurcation theory, stable and unstable manifolds, KAM theory, or the shadowing lemma. This course may also provide an introduction to a specific area of research, leading to possible PhD thesis projects.

110411 – Applied Analysis

Short Name:	ApplAnalysis
Type:	Lecture
Semester:	1
Credit Points:	7.5
Prerequisites:	None
Corequisites:	None
Tutorial:	No

Course contents The course Applied Analysis introduces a variety of analytical tools and methods which are used to model and analyse physical phenomena. Topics include: Fourier

transformations, partial and ordinary differential equations, operator theory, asymptotics (WKB, stationary phase, etc.), wavelets and applications.
Even though this courses covers the fundamentals of each of the subjects above, the emphasis will depend on the instructor. Students of applied mathematics or applied sciences are encouraged to participate in this course more than once.

110431 – Numerical Analysis

Short Name:	NumAnalysis
Type:	Lecture
Semester:	1
Credit Points:	7.5
Prerequisites:	None
Corequisites:	None
Tutorial:	No

Course contents This class is offered in two variants in alternating years. The first is focused on numerical linear algebra and optimization, the second is focused on differential boundary value problems and partial differential equations.

Topics in Variant A :

1. Matrix Caluclations: efficient solution of large sparse linear systems, acceleration techniques, preconditioning, and multigrid.
2. Nonlinear Equations: Newton's method, various forms of secant method, generalized linear methods, least squares methods, steepest descent, conjugate gradient and continuation methods.
3. Optimization: steepest decent, conjugate gradient, simulated annealing, Lagrangian methods, penalty methods.

Topics in Variant B :

1. Boundary Values Problems for ODEs: (multiple) shooting, collocation.
2. Finite Element and boundary element methods for PDEs
3. Spectral and Pseudospectral Schemes, time-stepping algorithms
4. Finite difference methods: consistency, stability; boundary conditions.

3.2 Reading Courses in Mathematics

Specialized topics, often related to faculty research areas, are taught in the form of reading courses. Offers depend on student and faculty interests.

3.3 Graduate Seminars

100591/100592 – Mathematics Colloquium

Short Name: MathColloquium
Type: Seminar
Semester: All
Credit Points: None
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents The weekly mathematics colloquium features talks by international scientists for the entire mathematical community, broadening horizons and encouraging formal or informal interactions.

100491/100492 – Graduate Research Seminar

Short Name: GradResearchSem
Type: Seminar
Semester: 1/2
Credit Points: 7.5
Prerequisites: None
Corequisites: None
Tutorial: No

Course contents This course is intended for beginning graduate students to help them identify interesting areas of research and possible thesis subjects and advisors. It consists of lectures mainly by professors, but also by other faculty, about current areas of research in mathemaical sciences, with particular emphasis on research areas of Jacobs faculty. Students get involved in discussions of all the areas of research; during the course of the semester, they choose at least three topics which they investigate further and which they elaborate into a research report. At the end of the semester, every student presents at least one of these reports. Participation is also open for advanced undergraduates looking for topics for their undergraduate theses, the results of which are presented as well.

Advanced Seminars

In addition, there are regular research seminars run by the faculty of the graduate program on advanced subjects and/or on topics of current research interests.

4 Qualifying Examination Syllabi

4.1 Algebra

Parts of the material covered in S. Lang, *Algebra*, Springer-Verlag.

Chapter I: Groups

1. Monoids
2. Groups
3. Cyclic groups
4. Normal subgroups
5. Operation of a group on a set
6. Sylow subgroups
7. Categories and functors (basic notions, know the language)
8. Direct sums and free abelian groups
9. Finitely generated abelian groups

Chapter II: Rings

1. Rings and homomorphisms
2. Commutative rings
3. Localization (at least know what it is)
4. Principal rings

Chapter III: Modules

1. Basic definitions (including exact sequences)
2. Homomorphisms
3. Direct products and sums of modules
4. Free modules

Chapter V: Polynomials

2. Definition of polynomials (OK if more concrete)
3. Elementary properties of polynomials
4. The euclidean algorithm
5. (Partial fractions - should be known from calculus)
6. Unique factorization in several variables
7. Criteria for irreducibility
8. The derivative and multiple roots
9. (Symmetric polynomials)
10. (Resultants, are useful, but not core)

Chapter VI: Noetherian Rings and Modules

1. Basic criteria
2. Hilbert's theorem
3. Power series

Chapter VII: Algebraic Extensions

1. Finite and algebraic extensions (know some examples, also of transcendental numbers)
2. Algebraic closure (existence)
3. Splitting fields and normal extensions
4. Separable extensions
5. Finite fields
6. Primitive elements
7. Purely inseparable extensions (at least know an example)

Chapter VIII: Galois Theory

1. Galois extensions
2. Examples and applications (something in this direction)
3. Roots of unity
4. (Linear independence of characters - more a technical tool)
5. The norm and trace
6. Cyclic extensions
7. Solvable and radical extensions. (Including the standard applications: regular n-gons, trisecting an angle, ...)

Chapter X: Transcendental Extensions

2. Hilbert's Nullstellensatz
3. Algebraic sets (have some idea about the relation between commutative algebra and geometry)

Chapter XVI: Multilinear Products

1. Tensor product
2. Basic properties
3. Extension of the base
4. Tensor product of algebras

4.2 Real Analysis

All references refer to G. Folland, *Real Analysis: Modern Techniques and Their Applications*, second edition, John Wiley & Sons.

1. **Measures**: algebras, sigma algebras, measures, outer measures, premeasures, Borel measures on the real line. Folland, Chapter 1
2. **Integration**: measurable functions, integration of real and complex valued functions, monotone and dominant convergence theorems, modes of convergence, product measures, Tonelli-Fubini theorem, the n-dimensional Lebesgue integral. Folland, Chapter 2
3. **Signed measures**: Hahn decomposition theorem, Jordan decomposition theorem, Radon Nikodym theorem, functions of bounded variation and absolutely continuous functions. Folland, Chapter 3.1, 3.2, 3.5
4. **Point set topology**: topological spaces, Urysohn's Lemma, Arzela-Ascoli theorem Folland, Chapter 4.1, 4.2, 4.3, 4.4, 4.6

5. **Elements of functional analysis**: Banach and Hilbert spaces, Hahn-Banach theorem, Baire Category theorem and consequences Folland, Chapter 5.1, 5.2, 5.3, 5.5
6. **Lp-spaces**: Minkowski's and Hoelder's inequality, bounded linear functionals on Lp Folland, Chapter 6.1, 6.2

4.3 Complex Analysis

The material covered in L. Ahlfors, *Complex Analysis*, third edition, McGraw–Hill.

4.4 Topology

The material covered in K. Jänich, *Topology*, Springer-Verlag.

4.5 Numerical Analysis

1. Basics of Error, Computational Work, and Stability Analysis
2. Numerical Linear Algebra (direct (Gauss/LU/QR) and iterative solvers (GS, ILU, PCG, GMRES), linear least-squares problems, SVD and eigenvalue solvers)
3. Numerical Methods of Analysis (integration, interpolation and approximation, FFT)
4. Nonlinear Equations and Optimization (Newton-type methods, nonlinear least-squares methods, global search methods, unconstrained and constrained smooth optimization) Initial Value Problems for ODE (implicit and explicit one- and multi-step methods, extrapolation methods, convergence and stability concepts, stiff problems,
5. *PDE discretization (finite difference and finite element method for elliptic and parabolic PDE, solution of sparse linear systems)
6. *Stochastic methods (pseudo-random numbers, MC simulations)

Basic Reading

1. A. Quarteroni, R. Sacco, F. Saleri, *Numerical Mathematics*, Springer-Verlag, 2000.

Alternative/Complementary Reading

1. J. Stoer, R. Bulirsch, *Introduction to Numerical Analysis*, third edition, Springer-Verlag, 2002.
2. G.H. Golub and C.F. Van Loan, *Matrix Computations*, third edition, Johns Hopkins, 1996.
3. W. Gautschi, *Numerical Analysis: An Introduction*, Birkhäuser, 1997.
4. E. Hairer, S.P. Norsett, G. Wanner, *Solving Ordinary Differential Equations I – Nonstiff Problems*, second edition, Springer-Verlag, 1993.
5. E. Hairer, G. Wanner, *Solving Ordinary Differential Equations II – Stiff and Differential-Algebraic Problems*, second edition, Springer-Verlag, 1996.
6. A. Quarteroni and A. Valli, *Numerical Approximation of Partial Differential Equations*, second edition, Springer-Verlag, 1997.
7. S. Larsson, V. Thomée, *Partial Differential Equations with Numerical Methods*, Springer-Verlag, 2003.

4.6 Partial Differential Equations

1. Linear model equations: Transport, Laplace, Heat, Wave Equations (classical solution techniques, representation formulas, energy methods)
2. First-Order Nonlinear Equations (method of characteristics, introduction to Hamilton-Jacobi equations and conservation laws)
3. Sobolev Spaces and elliptic boundary value problems (existence and regularity of weak solutions, weak and strong maximum principles)
4. Linear Evolution Equations (weak solutions of parabolic and hyperbolic equations, semi-group techniques)
5. *Variational and Nonvariational Methods for Nonlinear PDE (existence and regularity of minimizers and critical points of energy functionals associated with PDE)

Basic Reading

1. L.C. Evans, *Partial Differential Equations*, AMS, 1998 (Parts I, II and *Selected Topics from Part III)

Alternative Reading

1. J. Jost, *Partial Differential Equations*, Springer-Verlag, 2002.

4.7 Mathematical Physics

To be arranged with participating faculty.

4.8 Functional Analysis

The references *Conway, Folland and Rudin* refer to the three textbooks:

- *Functional Analysis, Walter Rudin, Second Edition,McGraw-Hill.*
- *A Course in Functional Analysis, John B. Conway, Second Edition, Springer.*
- *Real Analysis, Gerald Folland, Wiley.*

1. **Topological vector spaces and completeness** For example, Baire category theorem, open mapping theorem, closed graph Theorem. *Rudin, Chapter 1 and Chapter 2. Note: most of the material is also covered by the Real Analysis qualifier.*
2. **Convexity, weak topologies, duality in Banach spaces and compact operators** For example, Hahn-Banach theorem, Banach-Alaoglu theorem, Krein-Milman theorem *Rudin, Chapter 3 and Chapter 4.*
3. **Distributions and Fourier analysis and applications to differential equations** For example, Haar measure on compact groups (*Rudin, Chapter 4, pp 128-132*) and Fourier analysis on Groups (*Rudin, Fourier Analysis on Groups, Wiley, pages 1 to 13.*) Fourier transforms, Fourier series, distributions and tempered distributions, Sobolev spaces. *Folland, Chapter 8 and Chapter 9. Rudin, Chapter 6, Chapter 7 and Chapter 8.*
4. **Banach algebras and spectral theory** For example, Gelfand-Mazur theorem, commutative Banach algebras, Gelfand transforms, bounded operators on a Hilbert space, spectral theorem for normal and bounded operators. *Rudin, Chapter 10, Chapter 11 and Chapter 12.*

4.9 Probability Theory

1. General probability spaces. Discrete and geometric probabilities.
2. Random variables. Joint distribution function, density (if exists), quantiles. Examples, discrete and continuous (uniform, exponential, normal).
3. One-dimensional transformations of distributions.
4. Expectation of a random variable (and its function). Variance, moments.
5. Joint distributions and independence, marginal distributions, joint density.
6. Infinite sequences of random variables. Borel-Cantelli lemma. Modes of convergence. Convergence of expectations. Weak and strong laws of large numbers, central limit theorem.
7. Joint distributions: conditioning, correlation, and transformations. Conditional distributions, conditional expectation. Total probability formula (continuous case). Regression and correlation. Multidimensional transformations of distributions. Distribution and density of sum, product and quotient of one-dimensional random variables.
8. Countable Markov chains. Random walks.
9. Classification of finite Markov chains.